

Odd-even spin effects and variation of g factor in a quasi-one-dimensional subband

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Four odd-even spin phenomena are observed in electronic transport through a quantum wire. They consistently imply that a quasi-one-dimensional system tends to a spontaneous spin polarization with the energy band of minority spin-up electrons being reluctant to populate, thus widening the spin gap in energy. Variation of g factor within a single subband is measured using a dc conductance technique, showing clear evidence of an oscillatory g value as 1D subbands are filled one by one with increasing carrier density.

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Electron transport in one-dimensional (1D) quantum wires is of considerable interest due to possible applications in quantum logic¹ and the breakdown of Fermi-liquid theory.^{2,3} Despite the quantization of the ballistic conductance in units of $2e^2/h$, where the factor of 2 accounts for the spin degeneracy, being well established within a noninteracting Fermi-liquid picture, a number of spin-related phenomena in differential conductance, such as the 0.7 structure,⁴ the 0.7 analog,⁵ and other conductance anomalies under dc bias,⁶ cannot be understood in neither a noninteracting model nor the Luttinger-liquid theory.

Coulomb and exchange interactions are crucially important in quasi-one-dimensional (quasi-1D) systems as they could cause spontaneous spin polarization—one of the proposed interpretations of the 0.7 structure.^{7–15} The spin gap in energy is therefore expected to open, but how exactly this changes with carrier density is still not well understood. In a measurement of differential conductance a jump in conductance is observed whenever a particular one-dimensional subband crosses, first, the source potential and then the drain potential. This allows the spin gap and the g value to be measured at these two specific points but not as the subband moves between source and drain potentials. There is no theoretical consensus^{3,16} on whether spin symmetry can be broken in a 1D nanostructure, where there is lateral spread of wave function, as this is forbidden in an ideal 1D system. Consequently an investigation of the possible spin gap as the subband moves in energy would be of considerable value.

In this work, we present odd-even spin effects: (i) the conductance quantization value, (ii) the linear and (iii) non-linear transconductances, dG_{ac}/dV_g , and (iv) the gradient of the dc conductance, dG_{dc}/dV_g . These spin effects demonstrate how exactly the spin-down (lower energy) and the spin-up (higher energy) subbands behave when filled with electrons as the carrier density increases with split-gate voltage V_g . Briefly, the spin-up subband is relatively reluctant to populate, compared to the spin-down subband, and the spin gap thus widens with increasing carrier density. We demonstrate a method for directly tracking the spin gap and thus the effective g factor $|g^*|$ by which we show that the g factor *within a single 1D subband* is not constant but increases with carrier density. A simple simulation based on an analytical model with a widening spin gap gives good agreement with the observed odd-even spin effects.

Split-gate devices¹⁷ used in this work were fabricated on a GaAs/Al_{0.33}Ga_{0.67}As heterostructure. The two-dimensional

electron gas is 96 nm below the surface, with a low temperature mobility of 3.97×10^6 cm²/V s and a carrier density of 3.37×10^{11} cm⁻². Six devices with a width of 0.8 μ m and lengths from 0.3 to 1 μ m were measured and all exhibited similar characteristics. Differential conductance ($G_{ac}=dI/dV_{sd}$) and dc conductance ($G_{dc}=I/V_{sd}$) were measured simultaneously in a dilution refrigerator. In order to study the behavior of the spin-resolved subbands individually, a strong in-plane magnetic field was applied. By monitoring the Hall voltage, the out-of-plane misalignment was measured to be less than 0.3°. When examining the influence of the perpendicular component, we find that the magnetic length is several times longer than the corresponding wire width and the effects driven by it can be ignored.

Figure 1(a) shows the high magnetic field G_{ac} at various temperatures, exhibiting spin-resolved G_{ac} plateaux in multiples of e^2/h . As temperature increases up to ~ 1 K and no appreciable smearing of quantized plateau occurs, the quantized conductance of the first half-integer plateau, $0.5(2e^2/h)$, increases in value, whereas the conductance of the $2e^2/h$ plateau remains unaffected. This is in contrast to the expectation that the conductance of the spin-resolved quantized plateau should be e^2/h regardless of spin orientation. Figure 1(b) shows that this odd-even phenomenon in quantized conductance value was clearly observed up to the third 1D subband.

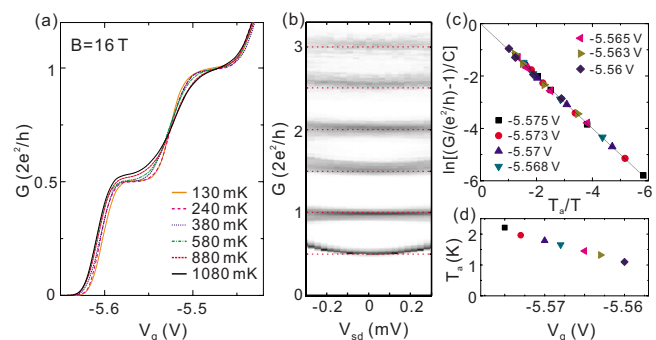


FIG. 1. (Color online) (a) Differential conductance (G_{ac}) at various temperatures in a magnetic field $B=16$ T. (b) Gray scale of transconductance (dG_{ac}/dV_g) versus V_{sd} and G_{ac} at $T=1080$ mK. Dark regions are plateaux with lower dG_{ac}/dV_g . The odd-even conductance values of the spin-split plateaux are shown up to the third 1D subband. (c) Temperature dependence of the conductance quantization showing the Arrhenius behavior at various gate voltages. (d) T_a obtained from the fits of G_{ac} to the Arrhenius equation.

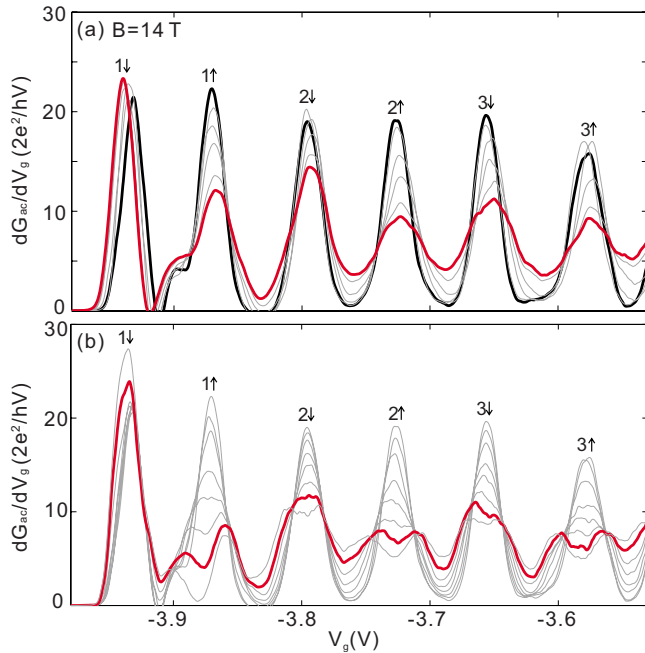


FIG. 2. (Color online) (a) Transconductance (dG_{ac}/dV_g) versus split-gate voltage at $B=14$ T for various temperatures: from top to bottom $T=130$ (black bold trace), 240, 380, 580, 880, 1080 (red bold trace) mK. (b) Transconductance versus V_g at $B=14$ T and $T=130$ mK for source-drain bias V_{sd} from 0 to 0.42 mV (top to bottom) in step of 0.06 mV. The transconductance at $V_{sd}=0.36$ mV is plotted with red bold line.

In previous work this spin-dependent odd-even conductance value of quantized plateaux was only observed in long wires ($\sim 5 \mu\text{m}$) and was attributed to a spin-dependent transmission probability.¹⁸ Here the temperature dependence measurements in short wires suggest that the increase of the half-integer quantized plateau in value results from the thermal population of the following spin-up energy level. Figure 1(c) shows that G_{ac} around the plateau is well fitted to the Arrhenius activation function, $G(T) = (e^2/h)[1 + C \exp(-T_a/T)]$, where T_a is the activation temperature. The activated behavior in the excess conductance above e^2/h is due to thermal population of the spin-up subband when it lies $k_B T_a$ above the chemical potential. We furthermore plot T_a against V_g in Fig. 1(d), showing an unusual behavior that the spin-up subband almost stops moving in energy with respect to the chemical potential as the carrier density increases. It moves by only $k_B T_a = 95 \mu\text{eV}$ over the range of the e^2/h plateau. The behavior that the spin-up subband almost stops populating is crucial for this odd-even spin effect; otherwise, the conductance would simply rise to $2e^2/h$ plateau if the spin-up subband keeps populating with carrier density. Note that the finding here is consistent with a recent theoretical prediction.¹³

Figure 2(a) shows another odd-even spin effect in the transconductance, dG_{ac}/dV_g . The spin-down transconductance peaks appear less temperature dependent than the spin-up peaks. With increasing temperature, the spin-up peaks decrease more rapidly than the spin-down peaks, and eventually the spin-up peaks become lower than the adjacent spin-down peaks. The transconductance peaks at

$T=1080$ mK (the red bold trace) clearly exhibit odd-even dG_{ac}/dV_g values.

A transconductance peak represents a subband passing through the chemical potential μ ; the height of the peak is determined by how quickly this occurs as a function of V_g . Since the thermal broadening of the subband levels is just determined by temperature, the odd-even dG_{ac}/dV_g peaks [Fig. 2(a)] therefore demonstrate that the spin-down subbands in general pass through the chemical potential faster than the spin-up subbands—consistent with the fact that the spin-down transconductance peaks are less sensitive to thermal broadening than the spin-up ones.

Further information is provided by the nonlinear differential conductance. The chemical potential splits into the source, μ_s , and the drain, μ_d , chemical potential when a source-drain bias V_{sd} is applied. The single peak in dG_{ac}/dV_g hence splits in two with increasing V_{sd} —the subband having to pass through μ_s and μ_d , respectively. In Fig. 2(b), we found the spin-up peaks split at lower V_{sd} compared to the spin-down peaks; for instance, at $V_{sd}=0.36$ mV [the red bold trace shown in Fig. 2(b)] all the first three spin-up peaks have clearly split in two while the spin-down peaks have not. This spin-dependent bias splitting further implies that the spin-down subbands are filled faster than the spin-up subbands. Indeed, the first spin-down branch does not exhibit splitting until a very large bias of $V_{sd}=1$ mV is applied.

In order to directly measure the rate at which the subband is filled with electrons, we performed the dc conductance measurements¹⁹ to probe how quickly the subband moves, between μ_s and μ_d , with respect to the chemical potential as a function of V_g . The dc conductance is given by $G_{dc} = (e^2/h)[n + \Delta E/eV_{sd}]$, where n is the number of subbands lying below μ_d and ΔE is the energy where a subband is occupied up to when it lies between μ_s and μ_d . One is therefore able to track the subband filling, ΔE , and consequently how quickly this changes, $d\Delta E/dV_g$, simply by measuring G_{dc} and dG_{dc}/dV_g .

Gray scales of both dG_{ac}/dV_g and dG_{dc}/dV_g versus V_g and V_{sd} are shown in Figs. 3(a) and 3(b), respectively. The branches in the dG_{ac}/dV_g gray scale [Fig. 3(a)] represent sharp increases in G_{ac} when a subband crosses either μ_s or μ_d , thus providing information about the energy of 1D subbands. In addition, the dG_{dc}/dV_g gray scale [Fig. 3(b)] quantitatively shows the rates at which a subband is filled as a function of V_g . We found an unexpected dc conductance plateau [marked $1\uparrow$ in Fig. 3(b)] in the region where the first spin-up subband lies between μ_s and μ_d , demonstrating that the spin-up subband almost stops populating as the carrier density increases. All the spin-up subbands lying between μ_s and μ_d exhibit a relatively lower dG_{dc}/dV_g compared to the spin-down subbands, showing that spin-up subbands are in general filled with electrons at a relatively lower rate as the carrier density increases with V_g .

Figure 4(a) shows that the modulus of the effective g factor, $|g^*|$, increases with V_g for the first three 1D subbands, where $|g^*|$ is obtained from the spin gap $|g^*|\mu B$ using the dc conductance technique (solid symbols) and the source-drain bias technique (open symbols). Note that the g factors obtained from these two nonequivalent methods are very consistent. The dc conductance method for determining $|g^*|$ is to

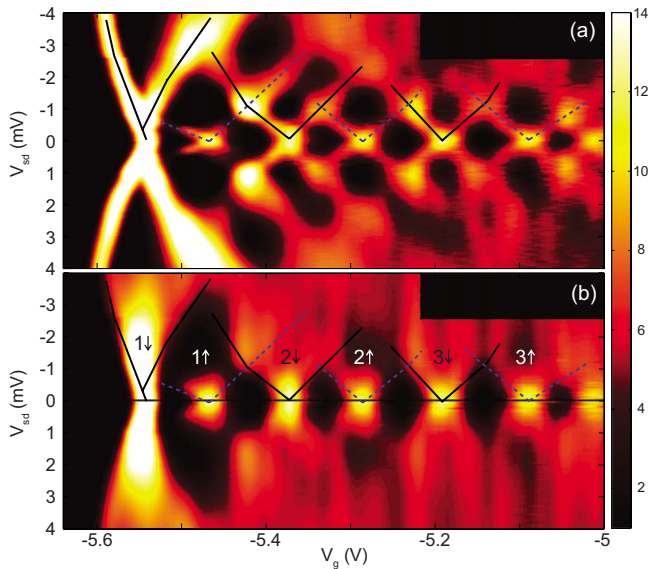


FIG. 3. (Color online) (a) Color-scale plot of dG_{ac}/dV_g as a function of V_g and dc bias at $B=16$ T and $T=130$ mK, showing where the spin-down (solid line) and the spin-up (dashed line) subbands pass through μ_s and μ_d . (b) Color-scale plot of dG_{dc}/dV_g which was measured simultaneously with dG_{ac}/dV_g shown in (a).

directly measure the subband filling ΔE by G_{dc} when the adjacent opposite-spin subband is aligned with either μ_s or μ_d , and consequently the spin gap $g^*\mu B$ is directly obtained by subtracting V_{sd} or the subband spacing $E_{n,m}$, as schematically shown in Fig. 4(b). This technique is more powerful than conventional techniques^{4,6,16} as it can quantitatively track the subband even after being populated; conventional techniques can only locate the energy of the subband when it just starts to populate. It therefore provides a method for studying variation of $|g^*|$ within a single subband as a function of V_g , whereas only one value of $|g^*|$ is measured for each subband by conventions.

We demonstrate in Fig. 4 that $|g^*|$ is enhanced compared to the bulk GaAs value of $|g^*|=0.44$ and, more important,

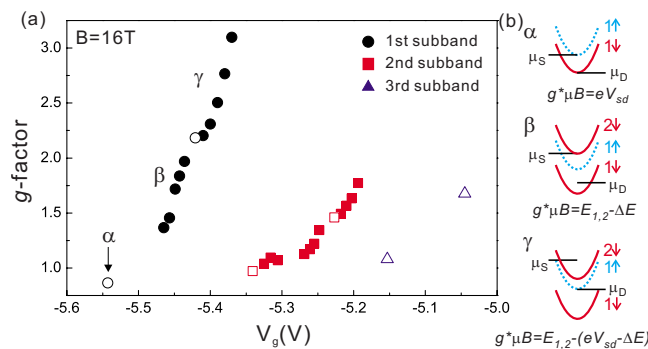


FIG. 4. (Color online) (a) The effective g factor as a function of split-gate voltage for the first three 1D subbands, as obtained from the Zeeman splitting at $B=16$ T. Solid symbols represent data obtained using the dc conductance method, while open symbols are data obtained by source-drain spectroscopy. (b) Schematics showing the configurations of subbands in three scenarios when the g factor in (a) is obtained.

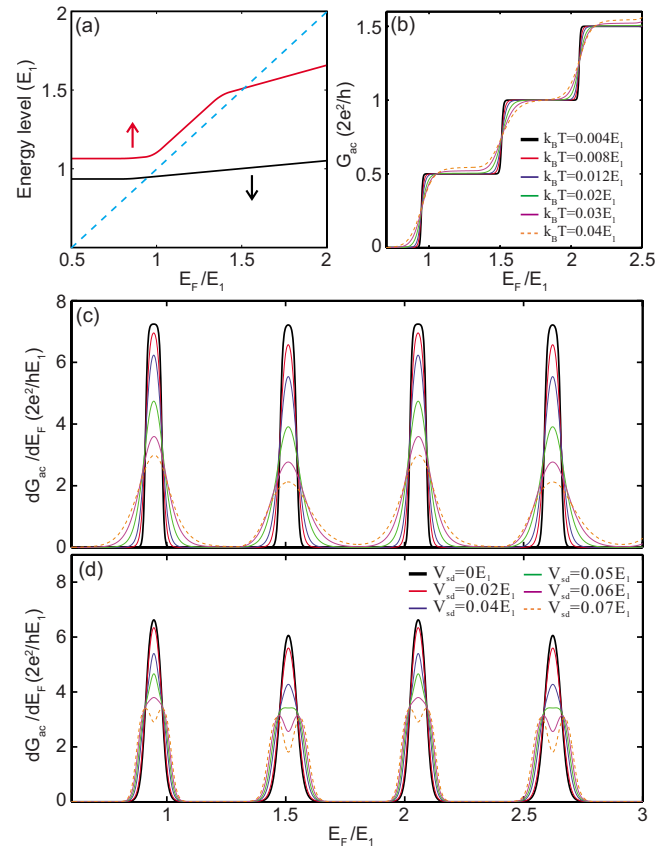


FIG. 5. (Color online) (a) Schematic of energy levels versus chemical potential μ . (b) Differential conductance for various temperatures from $k_B T = 0.004E_1$ (bold curve) to $k_B T = 0.04E_1$ (dashed). (c) Transconductance dG_{ac}/dE_F for the temperatures that are the same as (b). (d) Nonlinear transconductance dG_{ac}/dE_F for various source-drain bias from $0E_1$ (bold) to $0.07E_1$ (dashed) at $k_B T = 0.01E_1$.

continuously increases within a single subband as carrier density increases with V_g . The first measurable value $|g^*| \sim 1$ is close to the enhanced g values obtained in previous studies^{4,16} since then all $|g^*|$ were measured when the spin-up subband just starts to populate. It was previously found that $|g^*|$ decreases with increasing subband index⁴—it is the first measurable value for each subband that decreases. Here we go on to show that $|g^*|$ within a single subband increases with carrier density. However, the strength of the mechanism which enhances $|g^*|$ decreases as the subbands are populated consecutively with increasing carrier density.

It is important to stress that $|g^*|$ oscillates as the 1D subband is filled one by one. This is similar to the oscillatory $|g^*|$ in quantum Hall systems as the Landau level is filled one by one²⁰ and to the 1D theoretical prediction.⁷ Exchange interactions alone are predicted to cause a large spin splitting, but how the electron-electron interactions actually change the spin gap with carrier density is still poorly understood.^{7,11,13} Hence our findings assist knowledge of the interaction effects in 1D systems.

Simulations of electron transport, based on a simple analytical model,^{21,22} illustrate how these odd-even spin effects could occur. Here we assumed that the voltage drop is

symmetric,^{22,23} and the spin-down and spin-up energy levels populate as shown in Fig. 5(a), where the noninteracting first transverse energy level E_1 is used as the energy unit. Note that the functional forms of the 1D subbands [Fig. 5(a)] are very similar to a recent theoretical prediction.¹³ When the spin-up energy level is close to but still above μ , it moves very slowly with respect to the chemical potential μ . The energy gap is thus enhanced and no longer $|0.44\mu_B B|$ where 0.44 is the bulk value of $|g^*|$. The spin-up subband eventually starts populating but still in a relatively slow manner compared to the spin-down subband. Using the conjectured energy levels, we simulate the odd-even spin effects [Figs. 5(b)–5(d)] which are consistent with the experimental data. The subband spacing is measured to be 3.1 meV to estimate E_1 , giving agreement between the calculated and experimental results.

Various functional forms of the 1D energy levels are used to verify the simulated odd-even spin phenomena. We found the odd-even spin effects in transconductance [Figs. 5(c) and 5(d)] always occur whenever the spin-up subband passes through μ at a slower rate, no matter the precise functional

form of the movement. However, the spin effect in conductance plateaux [Fig. 5(b)] does require that the spin-up subband almost stops moving when it is close to μ . Without such an unusual behavior, the plateau would simply be washed out instead of increasing in value as the temperature is increased.

To summarize, we present four nonequivalent odd-even spin phenomena in transport measurements of quantum wires. In addition, we develop a dc conductance technique for measuring the variation of the spin gap *within a single subband*. An oscillatory g factor is observed as the 1D subbands are filled one after another with carrier density, providing the key to a fuller understanding of the electron-electron interactions in 1D systems.

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